$E = \sqrt{(pc)^2 + (m_o c^2)^2}$ $\qquad E = K + m_o c^2$

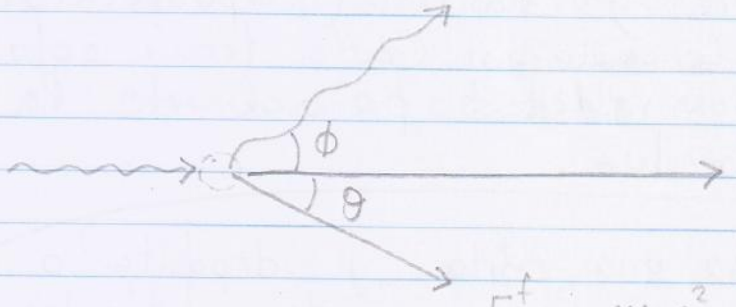
$$m = \frac{m_o}{\sqrt{1-(v/c)^2}}$$

$$E = mc^2 = \frac{m_o c^2}{\sqrt{1-(v/c)^2}}$$

$E^i_{fotón} = h\nu$

$P^i_{fotón} = \frac{h\nu}{c}$ $\qquad E^i_{e^-} = m_o c^2 \qquad P^i_{e^-} = 0$

$E^f_{fotón} = h\nu' \qquad \vec{P}^f_{fotón} = \left(\frac{h\nu'}{c}\cos\phi,\ \frac{h\nu'}{c}\operatorname{Sen}\phi\right)$



$E^f_{e^-} = m_o c^2 + K_{e^-}$

$\vec{p}^f_{e^-} = \left(p\cos\theta,\ -p\operatorname{Sen}\theta\right)$

Conservación momento lineal horizontal

$$P^i_{fotón} = \frac{h\nu}{c} = \frac{h\nu'}{c}\cos\phi + p\cos\theta \qquad (13)$$

Conserv. momento lineal vertical

$$0 = \frac{h\nu'}{c}\operatorname{Sen}\phi - p\operatorname{Sen}\theta \qquad (14)$$

$(13) \Rightarrow \quad pc\cos\theta = h\nu - h\nu'\cos\phi \qquad (15)$

$(14) \Rightarrow \quad pc\operatorname{Sen}\theta = h\nu'\operatorname{Sen}\phi \qquad (16)$

Elevando (15) y (16) al cuadrado

$$(pc)^2\cos^2\theta = (h\nu)^2 - 2(h\nu)(h\nu')\cos\phi + (h\nu')^2\cos^2\phi \qquad (17)$$

$$(pc)^2\operatorname{Sen}^2\theta = (h\nu')^2\operatorname{Sen}^2\phi \qquad (18)$$

Sumando (17) y (18)

$$\boxed{(pc)^2 = (h\nu)^2 - 2(h\nu)(h\nu')\cos\phi + (h\nu')^2 \qquad (19)}$$

Pero la energía del e^- es:

$$\boxed{E_{e^-} = \sqrt{(m_o c^2)^2 + p^2c^2} = m_o c^2 + K_{e^-}},$$

$$\Rightarrow \quad (m_o c^2)^2 + p^2c^2 = (m_o c^2 + K_{e^-})^2$$

$$(m_o c^2)^2 + p^2c^2 = (m_o c^2)^2 + K_{e^-}^2 + 2\, m_o c^2 K_{e^-}$$

$$p^2c^2 = K_{e^-}^2 + 2 m_o c^2 K_{e^-} \qquad (20)$$

Conservación de Energía $\Rightarrow$

$$\boxed{\begin{aligned} & h\nu + m_o c^2 = h\nu' + m_o c^2 + K_{e^-} \qquad (21) \\ \Rightarrow \quad & K_{e^-} = h\nu - h\nu' \qquad (22) \end{aligned}}$$

Sustituyendo (22) en (20)

$$p^2c^2 = (h\nu - h\nu')^2 + 2m_o c^2 (h\nu - h\nu')$$

$$\Rightarrow \quad \boxed{p^2c^2 = (h\nu)^2 - 2h\nu h\nu' + (h\nu')^2 + 2m_o c^2 (h\nu - h\nu') \qquad (23)}$$

Sustituyendo (23) en (19)

$$\cancel{(h\nu)^2} - \cancel{2h\nu h\nu'} + \cancel{(h\nu')^2} + 2\, m_o c^2 (h\nu - h\nu') = \cancel{(h\nu)^2} - 2\, h\nu h\nu' \cos\phi + \cancel{(h\nu')^2}$$

$$\Rightarrow \quad 2m_o c^2 (h\nu - h\nu') = 2h\nu h\nu' (1 - \cos\phi)$$

Dividir por $2h^2c^2$

$$\frac{m_0 c}{h}\left(\frac{\nu}{c}-\frac{\nu'}{c}\right)=\frac{\nu}{c}\frac{\nu'}{c}(1-\cos\phi)$$

Como $\nu=\frac{c}{\lambda}$ $\nu'=\frac{c}{\lambda'}$.

$$\frac{m_0 c}{h}\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)=\frac{1-\cos\phi}{\lambda\lambda'}.$$

$$\Rightarrow \lambda'-\lambda=\frac{h}{m_0 c}(1-\cos\phi) \qquad (24)$$

λ_c = longitud de onda Compton $= 2{,}426\times10^{-12}\, m = 2{,}426\, pm$

La variación de la longitud de onda $\lambda'-\lambda$ no depende de la longitud de onda λ incidente. Depende solamente de ϕ y de m_0.

$$(\lambda'-\lambda)_{max}=\frac{2h}{m_0 c}=4.852\, pm \qquad \phi=180°$$

$$\left(\frac{\lambda'-\lambda}{\lambda}\right)\times 100\% < 0.01\% \text{ para luz visible}$$

$$\sim 1\% \text{ para Rayos-X de } \lambda = 0.1\, nm$$

Experimento